

## AP Calculus BC Course Description

**COURSE OUTLINE:** The following topics define the AP Calculus BC course as it is taught over three trimesters, each consisting of twelve week grading periods.

### Limits and their Properties

- \* The Tangent Line Problem
- \* The Area Problem
- \* An Introduction to Limits
- \* Limits That Fail to Exist
- \* Properties of Limits
- \* Strategies for Finding Limits
  - Analytically
  - Graphically
  - Using tables to approximate limits
- \* Continuity at a point and one sided limits
- \* One-Sided Limits and Continuity on a Closed Interval
- \* The Squeeze Theorem for limits
- \* Properties of Continuity
- \* The Intermediate Value Theorem
- \* Definition of and determination of Infinite Limits and Vertical Asymptotes
  - Limits of Infinity
  - Limits at Infinity

### Differentiation

- \* Use graphing calculators: zoom-in exercise and local linearity
- \* The tangent line problem
- \* Definition of the Derivative of a function
- \* Sketching a Derivative Based on the Graph of  $f(x)$
- \* Relationship between differentiability and continuity
  - Zoom-in activity for ;
  - Sharp turn discussion
  - Vertical tangent discussion
- \* Differentiation Rules
  - Power Rule
  - Constant Multiple Rule
  - Sum and Difference Rule
- \* Derivatives of Sine and Cosine Functions
- \* Graphing only :  $f$  ,  $f'$  ,  $f''$
- \* Approximating rates of change from graphs and tables
- \* The product rule and quotient rule
- \* Derivatives of trigonometric functions
- \* Higher-Order Derivatives

- \* Position, Velocity, Acceleration Functions
- \* The Chain Rule
- \* The General Power Rule
- \* Trigonometric Functions and the Chain Rule
- \* Implicit Differentiation
- \* Related Rates

## Applications of Differentiations

- \* Extrema of a function
- \* Relative extrema and critical numbers
- \* Extrema on a closed interval
- \* Rolle's Theorem and the Mean Value Theorem
- \* Increasing and decreasing functions
- \* The First Derivative Test
  - Relative and absolute extrema
- \* The Second Derivative Test
  - Concavity
  - Points of inflection
- \* Horizontal asymptotes
- \* Curve-sketching techniques
  - Graphing given the functional equation
  - Relating  $f$ ,  $f'$ ,  $f''$
  - Symmetry of graphs
- \* Applied Minimum and Maximum Problems (Optimization)
- \* Calculating differentials
- \* Linear approximations
- \* Tangent line approximations
- \* Application problems involving position, velocity, and acceleration, and rectilinear motion
  - Graphical analysis of position versus time, velocity versus time, and acceleration versus time

## Integration

- \* Antiderivatives and indefinite integration
- \* Notation for anti-derivatives
- \* Basic Integration Rules
- \* Initial Conditions and Particular Solutions
- \* Area: Sigma Notation
- \* The Area of a Plane Region
- \* Definite integral as a limit of a Riemann Sum
  - Upper, Lower, Midpoint sums
  - Use Riemann sums and trapezoidal sums to approximate definite integrals presented analytically, graphically, or as tables of data.
- \* Trapezoidal Rule
- \* Definite Integrals and their properties

- \* Use of the First Fundamental Theorem of Calculus in evaluating integrals
- \* The Mean Value Theorem for Integrals
- \* Average Value of a Function
- \* The Second Fundamental Theorem of Calculus
- \* Integration by substitution; change of variables in the integration process
- \* The General Power Rule for integration
- \* Change of Variables for Definite Integrals
- \* Integration of even and odd Functions
- \* Functions defined by integrals
- \* Error Analysis

### **Logarithmic, Exponential, and other Transcendental Functions**

- \* The natural logarithmic function
- \* The number  $e$  as a limit
- \* The derivative of the natural log function
- \* Log rule for integration
- \* Integrals of trigonometric functions
- \* Inverse Functions
- \* Existence of an inverse function
- \* Derivative of an inverse function
- \* The natural exponential functions
- \* Derivatives of exponential functions
- \* Integrals of exponential functions
- \* Bases other than  $e$
- \* Applications of exponential functions
- \* Differential equations and separation of variables
- \* Growth and decay models
- \* Newton's Law of Cooling
- \* Slope Fields
  - Geometric interpretation of slope fields and differential equations
  - Drawing slope fields
- \* Euler's Method as a numerical solution of a differential equation
- \* Inverse trigonometric functions: Domains, ranges, and graphs
- \* Derivatives of inverse trigonometric functions
- \* Integration of inverse trigonometric functions

### **Applications of Integration**

- \* The integral defined as the accumulation of rates of change
- \* Area of a region between two curves
- \* Volume of solids of revolution
  - the disk and washer methods
  - the shell method
- \* Volume of solids with known cross sections
- \* Arc length in  $f(x)$  form
- \* Integration involved with motion applications of position, velocity, and acceleration.

Using initial conditions and the definite integral to calculate distance traveled.

### **Integration Techniques, L'Hopital's Rule, Improper Integrals**

- \* Review of basic integration methods
- \* Integration by parts : linear factors, tabular method
- \* Trigonometric integrals
- \* Integration by partial fractions
- \* Trigonometric substitution
- \* Powers of trig functions
- \* Logistic differential equations model
- \* L'Hopital's Rule and using it to determine limits
- \* Improper integrals and their convergence and divergence, with L'Hopital's Rule included

### **Sequences and Series**

- \* Definition of a sequence
- \* Convergence and divergence of sequences
- \* Definition of a series as a sequence of partial sums
- \* Partial sums as they pertain to convergence of a series
- \* Nth Term test for divergence
- \* Geometric Series Test for convergence and divergence
- \* Integral Test and P-Series Test: relate integral test to improper integrals and also to P-Series
- \* Direct Comparison Test , Limit Comparison Test
- \* Alternating Series Test, Alternating Series remainder  
Absolute and Conditional Convergence
- \* The Ratio and Root Tests
- \* Taylor polynomials and approximations: Use graphing calculator lab to introduce
- \* Power Series  
radius of convergence  
interval of convergence  
testing endpoints for convergence or divergence
- \* Taylor and Maclaurin Series for given functions
- \* Manipulation of series  
Addition of series, multiplication of series by constant or variable,  
differentiating a series, integration of a series
- \* Taylor's Theorem with the Lagrange Form of the Remainder

### **Plane Curves, Parametric Equations, and Polar Curves**

- \* Plane curves and parametric equations
- \* Graphing and orienting the path of a curve
- \* Finding first and second derivatives of functions in parametric form
- \* Parametric equations and vectors : motion along a curve, position, velocity, acceleration, speed, distance traveled
- \* Analysis of curves given in parametric and vector form
- \* Arc Length in parametric form

- \* Polar coordinates and polar graphs
- \* Arc Length in polar form
- \* Area bounded by a polar curve
- \* Area of a region formed by two polar curves